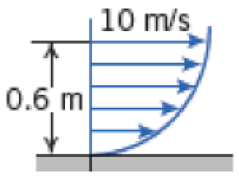
Chapter 1 – Introduction to Fluids Exercise Questions

1. The velocity profile for laminar flow between plates is given by:

$$\frac{u}{u_{max}} = 1 - \left(\frac{2y}{h}\right)^2$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider a flow of water at 15°C, with u_{max}= 0.1 m/s and h = 0.1mm. Calculate the shear stress on the upper plate.

2. Calculate the velocity gradient and shear stress for y = 0.1 m, y = 0.3 m and y = 0.6 m, if the velocity profile is a quarter circle with its centre at 0.6 m from the boundary and the maximum velocty is U = 10 m/s. The fluid viscosity is 7.5×10^{-4} N.s/m^2

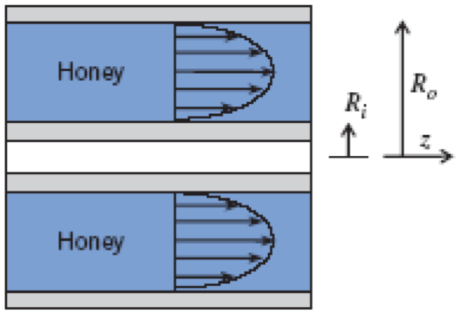


3. Crude oil at 20ºC fills the space between two concentric cylinders, which are 250 mm long and with diameters of 150 mm and 156 mm respectively. When the outer cylinder is fixed in place and therefore stationary, what torque is required to rotate the inner cylinder at 12rpm? The dynamic viscosity of crude oil at 20 ºC is 0.00718 Pa.s and neglect end effects.

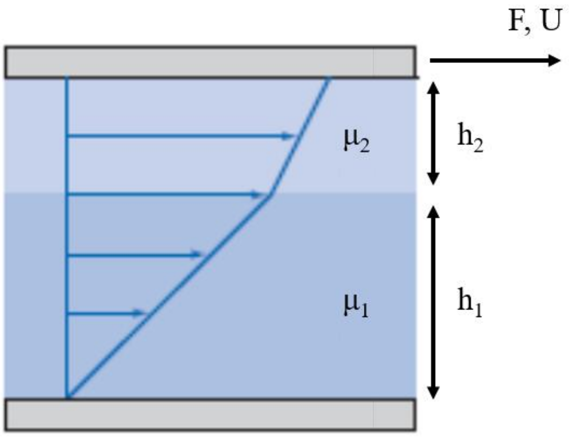
4. In a food processing plant, honey is pumped through an annular tube. The tube is L = 2 m long, with the inner radius of R_i = 5 mm and outer radius R_o = 25 mm. The applied pressure difference is Δp = 125 kPa and the honey viscosity is μ = 5 Ns/m^2. The theoretical velocity profile for laminar flow through an annulus is:

$$u_z(r) = \frac{1}{4\mu}\left(\frac{\Delta p}{L}\right)\left[R_i^2 - r^2 - \frac{R_o^2 - R_i^2}{ln\left(\frac{R_i}{R_o}\right)} ln\left(\frac{r}{R_i}\right)\right]$$

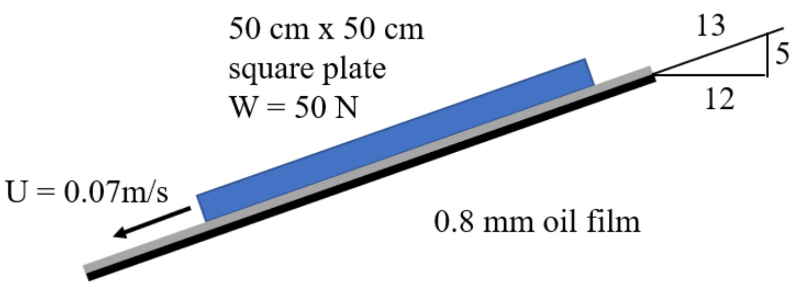
Show that the no slip condition is satisfied by this expression. Find the location at which the shear stress is zero. Find the viscous forces acting on the inner and outer surfaces and compare these to the pressure force $\Delta p\, \pi\left(R_o^2 - R_i^2\right)$.



5. Two layers of fluid are contained between two plates, each of $1m^2$ in area. The fluid viscosities are μ_1= 0.1 Ns/m^2 and μ_2= 0.15 Ns/m^2. The thickness of each layer of fluid is h_1 = 0.5 mm and h_2 = 0.3 mm. Find the force *F* that will make the upper plate move at a speed of *U* =1 m/s. What is the fluid velocity at the interface between the two fluids?



6. A plate weighing *W* = 50 N is placed on an inclined ramp with a thin oil film between them (film thickness *h* = 0.8 mm). The plate slides down the ramp at a speed of U = 0.07m/s, calculate the viscosity of the oil.



7. Waterjet cutters make use of very high pressure jets of water (and abrasives) to cut through hard materials such as steel. The pressure of these jets can reach up to 50 000 psi. At this pressure, how much volume change would you expect to see for a $1m^3$ volume of water (at atmospheric pressure)?

Exercise Solutions

1. Basic equation for shear stress:

$\tau = \mu \frac{du}{dy}$ where: $\frac{du}{dy} = \frac{d}{dy}\left(u_{max}\left[1 - \left(\frac{2y}{h}\right)^2\right]\right) = -\frac{8\,y\,u_{max}}{h^2}$

giving:

$\tau = -\frac{8\,y\,u_{max}}{h^2}\,\mu$

At the upper surface:

$y = \frac{h}{2}$

h = 0.1mm

u_{max} = 0.1 m/s

$\mu = 1.14 \times 10^{-3}$ N.s/m^2

Hence:

$$\tau = -\frac{8\,y\,u_{max}}{h^2}\,\mu = -\frac{8\,\frac{0.0001}{2}\,0.1}{0.0001^2}\,0.00114 = -4.56\,\frac{N}{m^2}$$

2. The equation for a quarter circle with y measured from the surface of the plate is:

$(x)^2 + (y - 0.6)^2 = 0.6^2$
$\Longrightarrow x^2 = 1.2y - y^2$

Knowing that at y = 0.6 m, U = 10 m/s gives:
$U^2 = 278(1.2y - y^2)$

Therefore, the velocity gradient and shear stress are:

$$\frac{dU}{dy} = \frac{139(1.2 - 2y)}{u}$$

$$\tau = \mu\frac{dU}{dy} = \mu\frac{139(1.2 - 2y)}{u} = 0.00075\frac{139(1.2 - 2y)}{u} = 0.104\frac{(1.2 - 2y)}{u}$$

So when y = 0.1 m

U = 5.53 m/s, $\frac{dU}{dy}$ = 25.14 1/s, τ = 0.019 N/m^2

and when y = 0.3 m

U = 8.66 m/s, $\frac{dU}{dy}$ = 9.63 1/s, τ = 0.007 N/m^2

and when y = 0.6 m

U = 10 m/s, $\frac{dU}{dy}$ = 0 1/s, τ = 0 N/m^2

3. Assume a linear velocity profile in the fluid between the two cylinders. First let us define the important parameters in the appropriate units:

   ω = 12 rpm = 1.257 rad/s

   dy = 0.003 m

   Using the equation for viscosity:

   $\tau = \mu \frac{du}{dy}$ where $u = \omega R_i = 1.257 \times 0.075 = 0.0942\ m/s$

   $$\tau = \mu \frac{du}{dy} = 0.00718 \frac{0.0942}{0.003} = 0.225\ N/m^2$$

   The torque is equal to the force required to move the inner cylinder multiplied by the radius of that cylinder. The force is equal to the shear stress and the surface area of the cylinder:

   $$F = \tau A = 0.225 \times (\pi \times 0.15 \times 0.25) = 0.0265N$$

   The torque is therefore:

   $$T = FR_i = 0.0265\ \times 0.075 = 0.002\ N.m$$

4. To check for the no slip condition is satisfied by the velocity profile, we look at the two walls and see what velocity is given:

   Hence, when $r = R_o$

   $$u_z(R_o) = \frac{1}{4\mu}\left(\frac{\Delta p}{L}\right)\left[R_i^2 - {R_o}^2 - \frac{R_o^2 - R_i^2}{ln\left(\frac{R_i}{R_o}\right)} ln\left(\frac{R_o}{R_i}\right)\right] = \frac{1}{4\mu}\left(\frac{\Delta p}{L}\right)\left[R_i^2 - {R_o}^2 + \left(R_o^2 - R_i^2\right)\right] = 0$$

   When $r = R_i$

   $$u_z(R_i) = \frac{1}{4\mu}\left(\frac{\Delta p}{L}\right)\left[R_i^2 - {R_i}^2 - \frac{R_o^2 - R_i^2}{ln\left(\frac{R_i}{R_o}\right)} ln\left(\frac{R_i}{R_i}\right)\right] = 0$$

   As we have zero velocity at the two walls, the no slip condition is satisfied.

   The shear stress is given as:

$$\tau = \mu \frac{du}{dr} = \mu \frac{d}{dr} u_z(r) = \mu \frac{d}{dr}\left[\frac{1}{4\mu}\left(\frac{\Delta p}{L}\right)\left[R_i^2 - r^2 - \frac{R_o^2 - R_i^2}{ln\left(\frac{R_i}{R_o}\right)} ln\left(\frac{r}{R_i}\right)\right]\right]$$

$$\tau = \frac{1}{4}\left(\frac{\Delta p}{L}\right)\left[-2r - \frac{R_o^2 - R_i^2}{ln\left(\frac{R_i}{R_o}\right) r}\right]$$

The location of zero shear stress is:

$$0 = -2r - \frac{R_o^2 - R_i^2}{ln\left(\frac{R_i}{R_o}\right) r} \quad \Rightarrow \quad r = \sqrt{\frac{R_i^2 - R_o^2}{2ln\left(\frac{R_i}{R_o}\right)}} = \sqrt{\frac{0.005^2 - 0.025^2}{2ln\left(\frac{0.005}{0.025}\right)}} = 13.7mm$$

Find the viscous forces on the inner and outer surfaces. This can be found my multiplying the shear stress by the surface area.

$$F = \tau A = \frac{1}{4}\left(\frac{\Delta p}{L}\right)\left[-2r - \frac{R_o^2 - R_i^2}{ln\left(\frac{R_i}{R_o}\right) r}\right] 2\pi R_o L$$

The force on the outer surface is:

$$F_o = \Delta p\pi \left[-R_o^2 - \frac{R_o^2 - R_i^2}{2\ ln\left(\frac{R_i}{R_o}\right)}\right] = -172.2\ N$$

The force on the inner surface is:

$$F_i = \Delta p\pi \left[-R_i^2 - \frac{R_o^2 - R_i^2}{2\ ln\left(\frac{R_i}{R_o}\right)}\right] = 63.4\ N$$

Therefore, the total viscous forces are:

$$F_o - F_i = -172.2 - 63.4 = -235.6\ N$$

Comparing this to the pressure force:

$\Delta p\ \pi\left(R_o^2 - R_i^2\right) = 125000\ \ \pi(0.025^2 - 0.005^2) = 235.6\ N$

We can see that the pressure force is equal to the total viscous forces.

5. The shear stress is the same throughout (the velocity gradients are linear, and the stresses in the fluid at the interface must be equal and opposite).

$$\tau = \mu_1 \frac{du_1}{dy_1} = \mu_2 \frac{du_2}{dy_2} \quad \Longrightarrow \quad \mu_1 \frac{U_i}{h_1} = \mu_2 \frac{(U - U_i)}{h_2}$$

Solving for the interface velocity U_i

$$U_i = \frac{U}{1 + \frac{\mu_1}{\mu_2}\frac{h_2}{h_1}} = \frac{1}{1 + \frac{0.1}{0.15}\frac{0.3}{0.5}} = 0.714\ m/s$$

The force required is:

$$F = \tau A = \mu_1 \frac{U_i}{h_1} A = 0.1 \times \frac{0.714}{0.5} \times 1 = 143\ N$$

6. Assume a linear velocity profile. The drag force caused by the viscosity of the oil must equal the component of weight along the surface.

   $F = W\ sin\theta = \frac{5}{13} W$

   The force due to viscosity is:

$$F = \tau A = \mu \frac{du}{dy} A = \mu \frac{U}{h} A$$

   Equating the two forces:

$$\frac{5}{13} W = \mu \frac{U}{h} A$$

$$\mu = \frac{h}{AU}\frac{5}{13} W = \frac{0.0008}{0.25 \times 0.07} \times \frac{5}{13} \times 50 = 0.879\ Pa.s$$

7. The modulus of elasticity for water is:

$$E_v = -\Delta p \frac{V}{\Delta V}$$

   Converting 50 000 psi to Pascal gives $\Delta p = 344.74 \times 10^6\ Pa$

$$\Rightarrow \Delta V = -\Delta p \frac{V}{E_v} = -(344.74 \times 10^6) \frac{1}{(2.2 \times 10^9)} = -0.1567 m^3$$

   So the final volume of the water is:

$$V_{final} = V + \Delta V = 1 - 0.1567 = 0.8433\ m^3$$