UNIVERSITY OF TORONTO, FACULTY OF APPLIED SCIENCE AND ENGINEERING

MAT187H1S – Calculus II – Final Exam - April 20, 2018

EXAMINERS: G. CHEN, S. COHEN, B. GALVÃO-SOUSA, K. MATETSKI, D. PANCHENKO, F. PARSCH

Time allotted: 150 minutes

No Aids permitted

Total marks: 80

First name (please write as legibly as possible within the boxes)

Last name

Student Number

Instructions:

- Do not write on the QR code at the top of the pages.
- This test contains 12 pages and a detached booklet for multiple-choice questions and formula sheet. DO NOT DETACH ANY PAGES.
- You can use pages 9–11 to complete questions (**mark clearly** which questions you are answering).
- Calculators, cellphones, or any other electronic devices are not allowed. If you have a cellphone with you, it must be turned off and in a bag underneath your chair.
- DO NOT START the test until instructed to do so.

GOOD LUCK!

LONG ANSWER PART

11. A railroad company plans to lay railroad tracks on a flat plain. The tracks can be modelled as a curve $y = f(x)$ for some function $f(x)$. You can assume $z = 0$. **(20 marks)**

Note. You can use the results of previous parts even if you didn't solve them.

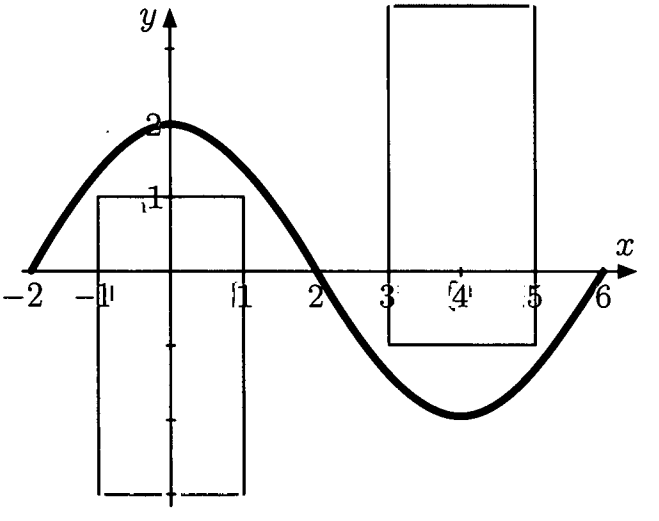
(a) (7 marks) Show that the curvature of the track at a point $\big(x, f(x), 0\big)$ is

$$\kappa = \frac{|f''(x)|}{\big(1 + f'(x)^2\big)^{\frac{3}{2}}} \quad .$$

Hint. Calculations are easier with the cross product formula.

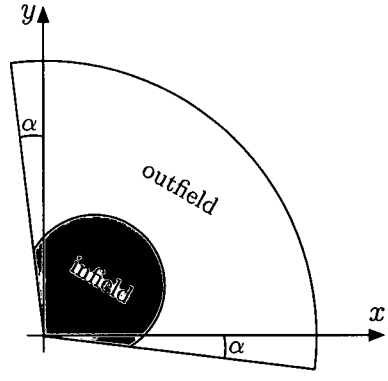
(b) (3 marks) Show that $\kappa \leqslant |f''(x)|$.

(c) (4 marks) To bypass two farms in the path, between $-2 \leqslant x \leqslant 6$ the track goes along the curve $y = a\cos(bx)$ shown in the figure for some $a > 0$ and $b > 0$. What are a and b?



(d) (6 marks) The maximum curvature of the track must not exceed 2. Is the track in **(c)** acceptable? Justify your answer.

12. Consider the irregular baseball field in the figure. **(20 marks)**



The field is bounded by lines with angles $-\alpha$ and $\pi/2 + \alpha$ for some small angle α.

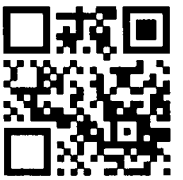
The curve that separates the outfield and the infield is given by a circle of radius $\sqrt{2}$ centred at the point $(1, 1)$.

(a) (5 marks) Write a formula for the circle in x-y-coordinates.

(b) (5 marks) Write the formula $r = f(\theta)$ for the circle in polar coordinates. Justify your answer.

Hint. Use the formulas $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

$r =$



(c) **(10 marks)** Compute the infield area. Justify your answer.

Area =

13. Consider the differential equation **(20 marks)**

$$y'' - 5y' + 4y = 10\cos(2x)$$

and the initial conditions

$$y(0) = 3 \qquad \text{and} \qquad y'(0) = 10.$$

(a) (5 marks) Find the Taylor polynomial $p_2(x) = c_0 + c_1 x + c_2 x^2$ of degree $n = 2$ centred at $a = 0$ for the solution $y(x)$ of the above equation **without** solving the differential equation. Justify your answer. (You will be asked to solve the differential equation in part **(b)** below.)

$p_2(x) =$

(b) (8 marks) Find the solution of the initial value problem. Justify your answer.

$y(x) =$

(c) (5 marks) Find the Taylor series centred at $a = 0$ of the solution found in **(b)**. Justify your answer.

(d) (2 marks) Find the coefficient of x^4 of the Taylor series you found in **(c)**.

$c_4 =$

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If you wish to have this page marked, make sure to refer to it in your original solution.

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