Faculty of Applied Science and Engineering
University of Toronto

MAT 186 H1S - CALCULUS I
TUESDAY, APRIL 16, 2019

FINAL EXAMINATION

FAMILY NAME: _____

GIVEN NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Time allowed: 2 hours, 30 minutes
Total marks: 80
No calculators allowed.
Examiner: S. Cohen
Solutions will be marked for both correctness **and** clarity.
Use the backs of pages when necessary,
indicating clearly where solutions continue.

FOR MARKER'S USE ONLY	
QUESTION	MARK
1	/ 20
2	/ 10
3	/ 20
4	/ 10
5	/ 10
6	/ 10
TOTAL	/ 80

1. Some warm-up questions: **Find the following**. Justify your answers fully.

$$\lim_{x \to -2} \frac{x^3 + 2x + 12}{3x^2 + 4x - 4}$$

[3 marks for a good solution, 1.5 if you resort to L'Hopital]

The tangent to $\sin(xy) + 4 = 2x - y$ at $(2,0)$.

[4 marks]

$$\lim_{x\to\infty} x^{1/x}$$

[5 marks]

$$\lim_{x\to 2}(x-2)\cos\left(\frac{1}{(x-2)^2}\right).$$

[5 marks]

$$\int_{-1}^{0} -2t\cdot e^{4t^2-3}\,dt$$

[3 marks]

2. **Use Riemann sums** to evaluate:

$$\int_{-1}^{3} 6t^2 - 4t\, dt$$

You may find the following useful:

$$\sum_{k=1}^{n} 1 = n, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

[10 marks]

3. Graph the function $f(x) = -2x - 3 + \frac{2}{|x-1|}$.

- Organize your solution well.
- Include all of the important values, **including any easy-to-find roots** and identify their type (maximum, minimum, etc.).
- Indicate regions where the function is increasing vs. decreasing, concave up vs. concave down.
- You have two pages for this problem.

[20 marks]

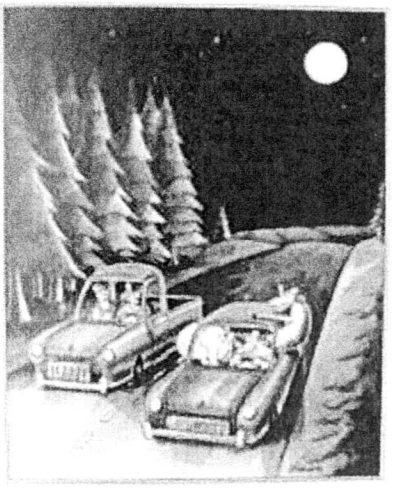
[Continue Question 3 on this page.]

4. One ship, *The Ovine*, leaves its harbor at 9 pm. It travels east at 40 kph. At the moment it leaves, a second ship, *The Hapless*, is $100\sqrt{2}$ km to the southeast and travelling at 30 kph north.

   (a) How fast are the ships approaching each other at 10 pm?

   (b) At what time are the ships at their closest to each other?

[10 marks]



Sheep that pass in the night

5. Milkshake is poured into a conical glass with a radius of 5 cm at its top and a height of 9 cm. You and your friend decide to split the milkshake – using a straw (set at a constant height above the glass) one of you drinks half of the milkshake (by volume) and then the other drinks the rest. If you wish to minimize your work, should you drink first or second?

   Note that the volume of a cone is $\frac{\pi r^2 h}{3}$.

[10 marks]

6. Let a be a parameter with $0 \leq a \leq 6$. A triangle with the vertices $(0,0)$, $(a, 6-a)$, and $(2a, 0)$ is rotated about the $x-$ axis. What is the greatest possible volume for the shape created?

[10 marks]

[Extra page for additional work.]

[0 marks]