

**CIV 401: Design of Wind and Hydro Plants**

**Department of Civil Engineering  
University of Toronto**

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**Final Exam: 20 December 2019 (2.5 hours; Type C)**

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**INSTRUCTIONS**

1. In clear responses, answer four (4) of the following six (6) equally-weighted questions. **Questions will be evaluated for their clarity, accuracy and completeness.**
  2. Unless otherwise stated, assume that all data is given in consistent SI units and that the working fluid is water. If doubt exists as to the interpretation of a question, a clear statement of any assumptions made should be included with the answer. Tests are exercises in communication: answer questions clearly and legibly.
  3. Two pages summarizing various hydraulic formulae and relations are also distributed. Silent, non-printing, non-communicating calculators are permitted. A single 8.5 by 11 inch aid sheet is also permitted (both sides can be used). This exam paper consists of a **total of 10 pages**. Answers can be written in the blank spaces or on the back of any page, but indicate clearly at the top of each back page what question is being answered.
  4. If any question involves iteration, 75% of the assigned grade will be awarded to a correct first iteration including application of the determined corrections.
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1. Assume you are developing the wind resources in a particular area and the prevailing wind has an average velocity of between 12 and 15 m/s. Explain how you would size a single wind turbine to achieve a 5 MW rated capacity. Explain also how the operational efficiency of the turbine would change over the full range of wind expected at the site (assumed to vary, say, from 0 to 50 m/s).

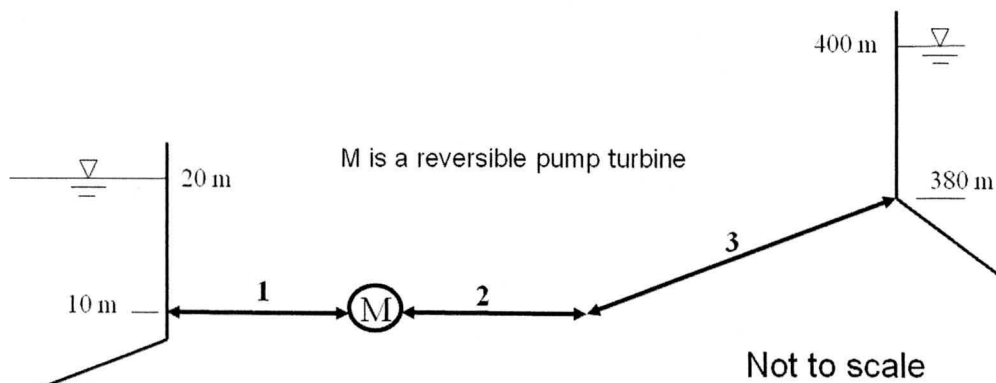
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2. The following quotation is taken from Vogel's *Cat's Paws and Catapults*. It nicely describes pumping in qualitative terms. Illustrate as precisely as you can, through sketches and equations how you would now, as a system designer, interpret this quotation quantitatively.

What does a pump – any pump – do? It uses power to raise the pressure of fluid flowing through it. So three things matter: power output, increase in pressure, and rate of flow. Power is the increase in pressure times how fast the fluid flows (volume flowing per time, not the speed of a bit of fluid). Put another way, a pump makes fluid go through some system (load) that it wouldn't go through by itself. If the load resistance is high, as when fluid has to flow through a long and skinny pipe, then a lot of pressure produces only a modest trickle. Conversely, if the load resistance is low – as with a short, fat pipe – a little pressure gives a great gush of fluid. Obviously a pump needs sufficient power to do its intended job. A bit less obviously a pump ought to be appropriately matched to the resistance of its intended load.

3. When the flow in a system is suddenly adjusted, a pressure wave is created that propagates through the connected pipe system. What makes this pressure significant is that it is of large magnitude and propagates quickly. Explain, with specific reference to turbine system, what makes the system most sensitive or vulnerable to this pressure wave and what procedures or approaches are available to system operators to limit the magnitude of the resulting water hammer wave. Is the compressibility of the system a benefit or an even greater danger to these concerns?

4. In the pumped storage system sketched below, water is passed between two large storage reservoirs. The water is either pumped uphill or the machine (M) is run as turbine to generate power. For the design conditions shown, assume that both the pump and the turbine operate with a conveyance velocity of 3.00 m/s and that they each have a 90% average efficiency. (a) Estimate the power required from the electrical system when it is used in pumping mode and (b) the electrical power delivered to the system when operated in turbine mode. (c) Estimate also how much additional power would be available for useful work if the third (and longest conduit) were to be twinned? That is, if an identical 2500 m conduit were added in parallel to the original 3<sup>rd</sup> pipe, assuming the overall discharge was not increased. (d) Explain whether you think the pumping or turbine mode is likely to have the greatest control on the design and why. Finally, estimate the maximum power that could be extracted from this system.



$$L_1 = L_2 = 500 \text{ m}; \quad L_3 = 2500 \text{ m}$$

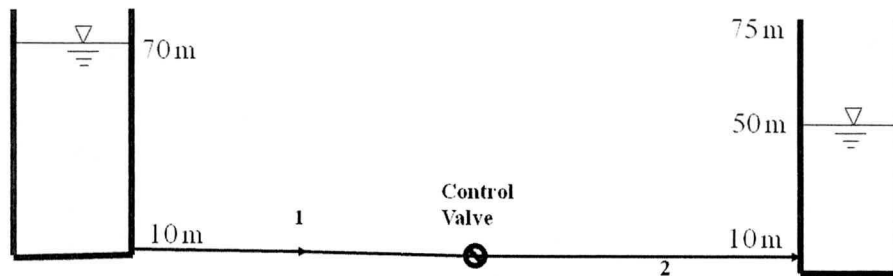
$$D_1 = D_2 = D_3 = 2.000 \text{ m}$$

$$f_1 = f_3 = 0.015; \quad f_2 = 0.024$$

Assume efficiency of machine – acting either as a pump or a turbine – is 90%; assume the design velocity is 3.00 m/s

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5. For the two-pipe and two-tank system sketched below, at a particular initial condition, the head in the upstream tank is 70 m and the head in the downstream tank is 50 m as shown; the upstream tank has a cross sectional area of  $50 \text{ m}^2$  and the downstream tank has an area of  $20 \text{ m}^2$ . Both pipes are 1500 m in length, have a diameter of exactly 0.6 m, and an estimated Darcy Weisbach friction factor of 0.028. The control valve at the midpoint dissipates 5 velocity heads when fully open. (a) For the initial condition shown, estimate the velocity (in m/s) and quasi-steady discharge in cubic meters per second. (b) If the valve's is left fully open, describe as accurately as you can what will happen as the system moves from its initial condition to a final equilibrium; the final equilibrium should be fully specified. (c) Specify mathematically both a quasi-steady and a rigid water column model, so that the system operator could determine how the system will evolve from its initial to its final condition; assume the control valve opening might change over the operation and thus be a function of time. Which model would you recommend using? (If you need additional information about the system, specify what else you'd need to know to make a reasonably accurate specification of the system's evolution.)



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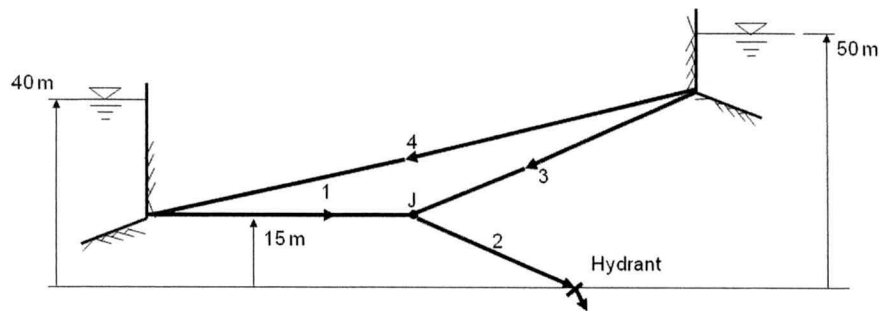
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6. In the system below, the following values are provided (f is the Darcy-Weisbach friction factor)

Pipe	f	L (m)	Dia. (m)
1	0.012	1500	0.381
2	0.018	800	0.533
3	0.015	2000	0.457
4	0.025	3000	0.381

For this system, write a detailed rigid water column model to represent the system dynamics associated with opening the hydrant. Assume that the hydrant represents the only local loss in the system and the dynamic state is created by increasing the relative gate opening (i.e., the “tau” value) of the hydrant from a fully closed position ( $\tau = 0$  initially). Assume also that the hydrant has a fully open loss coefficient of 4 velocity heads. Discuss the key assumptions you are making to create your model and also how you’d numerically simulate the system’s response. What key factors would determine whether a simpler quasi steady model might have been used instead of a rigid column model? What concerns would likely motivate the creation of such a simulation model?



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## Pipe Relations

Assumed commercially available pipe sizes:

ACTUAL DIAMETER (inches)	ACTUAL DIAMETER (mm)	NOMINAL DIAMETER (mm)	ACTUAL DIAMETER (inches)	ACTUAL DIAMETER (mm)	NOMINAL DIAMETER (mm)
6	152	150	42	1067	1100
8	203	200	48	1219	1200
10	254	250	54	1372	1400
12	305	300	60	1524	1500
15	381	400	66	1676	1700
18	457	450	72	1829	1800
21	533	500	78	1981	2000
24	610	600	84	2134	2100
27	686	700	90	2286	2300
30	762	800	96	2438	2400
36	914	900	102	2591	2600

**Bernoulli equation:**  $\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f$

**Darcy-Weisbach:**  $h_f = f \frac{L}{D} \frac{v^2}{2g} = 0.0826 \frac{Q^2}{D^5} L f$

**Laminar:**  $f = \frac{64}{Re}$       **Swamee-Jain:**  $f = \frac{0.25}{\left[ \log \left( \frac{e}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right]^2}$

**Hazen-Williams (SI):**  $Q = 0.278CD^{2.63}S^{0.54}$ , where energy slope  $S = h_f/L$

**Conductance:**  $Q = E[\Delta H]^m$       **Resistance:**  $\Delta H = K[Q]^n$

**Conductance:**  $E = \left( \frac{1}{K} \right)^{1/n}$       **Resistance:**  $K = \left( \frac{1}{E} \right)^{1/m}$

Relations (SI)	$m$	$n$	$E$	$K$
Hazen-Williams	0.54	1/0.54	$0.278CD^{2.63}/L^{0.54}$	$L/(0.278CD^{2.63})^{1/0.54}$
Darcy-Weisbach	0.5	2	$D^{2.5}/(0.0826fL)^{0.5}$	$0.0826fL/D^5$

**Pipes in Series:** Add resistances.

**Pipes in Parallel:** Add conductances.

## Some Key Concepts and Properties

Numerical values in the following table are typical (but not universal) of municipal practice.

Property	Units/Magnitudes	Relations
Density, $\rho$	1000 kg/m <sup>3</sup>	$\rho = M/V$
Specific weight, $\gamma$	9800 N/m <sup>3</sup>	$\gamma = \rho g$
Velocity, $v$	$\leq 4$ m/s	$v = dx/dt$
Discharge, $Q$	m <sup>3</sup> /s, l/s	$Q = vA$
Mass flux, $\dot{m}$	kg/s	$\dot{m} = \rho Q$
Shear stress, $\tau$	N/m <sup>2</sup>	$\tau = F_t/A$
Dynamic Viscosity, $\mu$	10 <sup>-3</sup> N s/m <sup>2</sup>	$\tau = -\mu \frac{dv}{dy}$
Kinematic Viscosity, $\nu$	10 <sup>-6</sup> m <sup>2</sup> /s	$\nu = \mu/\rho$
Reynolds Number	Dimensionless (< 2000 laminar)	$Re = vD/\nu$
Gauge Pressure, $P$	N/m <sup>2</sup> (rel. to atmosphere)	$P = F_n/A$
Absolute Pressure, $P_a$	N/m <sup>2</sup> (rel. to vacuum)	$P = F_n/A$
Pressure Head, $h_{pr}$	m	$h_{pr} = P/\gamma$
Velocity Head, $h_{vel}$	m	$h_{vel} = v^2/2g$
Elevation Head, $z$	m	vertical distance above datum
Piezometric Head, $h$	m (or hydraulic head)	$h = P/\gamma + z$
Total Head, $H$	m	$H = h + v^2/2g$
Bulk Modulus, $K$	2.07 · 10 <sup>9</sup> Pa	$K = \frac{\Delta P}{\Delta \rho/\rho}$
Specific Heat, $C$	4200 J/(kg C)	energy-temperature relation

**Continuity:** Mass of water is conserved.

1. Steady flow in single or series pipe:  $Q = \text{constant}$
2. Junction: Sum of inflow = sum of outflow:  $\sum Q_{in} = \sum Q_{out}$
3. Tank or Reservoir:  $\frac{dS}{dt} = \sum Q_{in} - \sum Q_{out}$

**Energy:** Bernoulli equation:  $\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f$

**Losses:**  $h_f$  complex; with Darcy-Weisbach:  $h_f = f \frac{L}{D} \frac{v^2}{2g}$