ECE 110 - Exam Jam

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Preparing for the exam

- Develop a study plan or timetable
- Decide which topics you need to spend most time on
- Review Wiley Plus examples, and midterm papers
- Look at old exam papers

The Final Exam

- The Final Assessment will cover all materials listed in the course syllabus.
- The Final Assessment questions will be taken from the WileyPlus question pool and administered on WileyPlus (similar to the Assignments and Term Test-1 and Term Test-2)
- There will be 7 questions on the Final Assessment, and students will receive similar questions
  - One question on electrostatics
  - Two questions on magnetism
  - Two questions on DC circuit analysis
  - One question on the first-order transient circuit
  - One question on AC circuit analysis

Calculators

- FASE approved Calculators are:
  - Casio fx-991 MS and
  - Sharp EL-520X
- Make sure you know how to use them
  - Simultaneous Equations (2x2) and (3x3)
  - Physical constants - $\varepsilon_0 \ and \ \mu_0$
  - Complex numbers – addition, subtraction, multiplication and division, polar – rectangular conversion
- Can you solve:
  - $(4k + 6k)I_1 - 6\text{k}I_3 = -6$
  - $(9k + 3k)I_2 - 3kI_3 = 6$
  - $-6kI_1 - 3kI_2 + (3k + 6k + 12k)I_3 = 0$
- And
  - $I = \frac{V}{Z} = \frac{12\sqrt{2}\angle 90^o}{4\sqrt{2}\angle 45^o}$

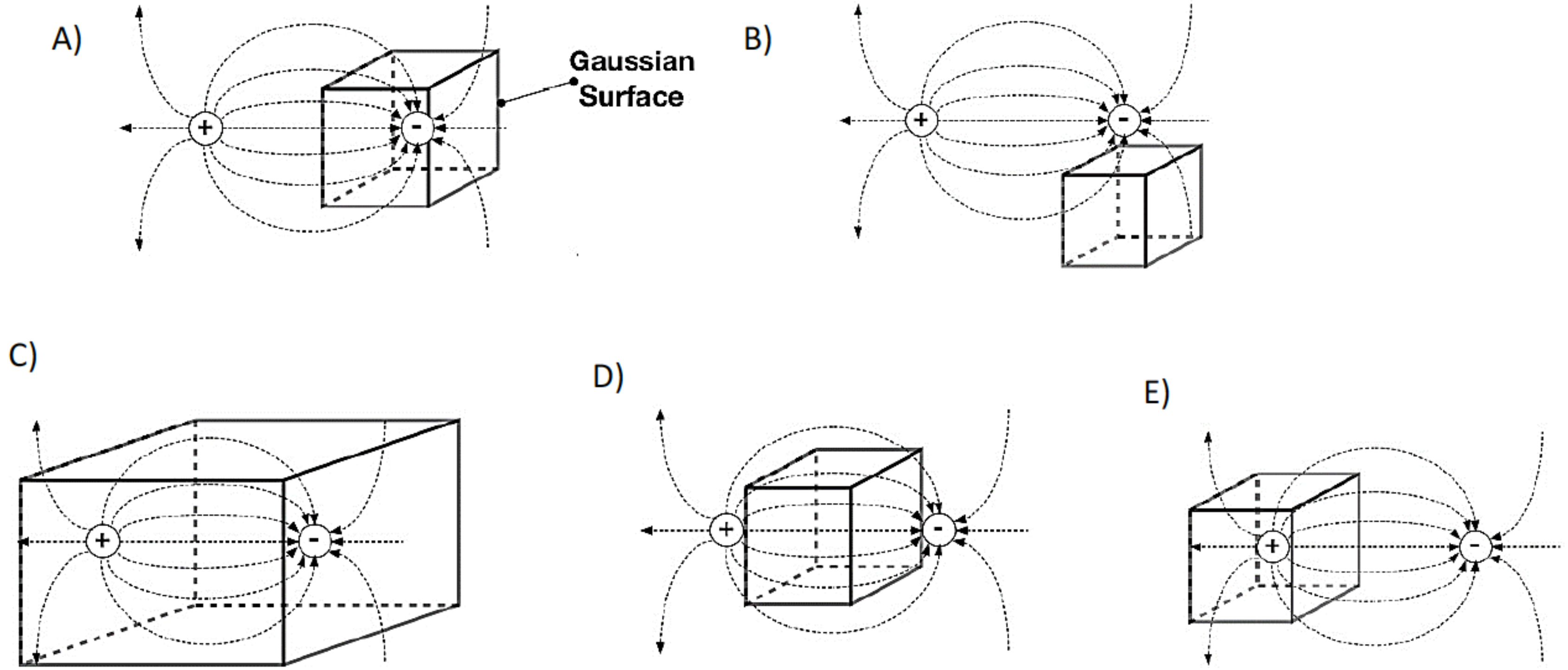
Topics covered in ECE 110

- Electrostatics, Gauss’s Law, electric potential, Capacitors and Resistors
- Magnetic Fields, and Ampere’s Law and Faraday’s Law, Inductors
- DC circuits
- First Order Circuits
- AC Circuits

Gauss' Law

- Relates the electric flux through a closed surface to the charge enclosed by that surface.
- Key Ideas:
  - $\Phi = \oint \overrightarrow{E}.d\vec{A}$
  - Both $\vec{E}$ and $d\vec{A}$ are vectors.

Gauss' Law



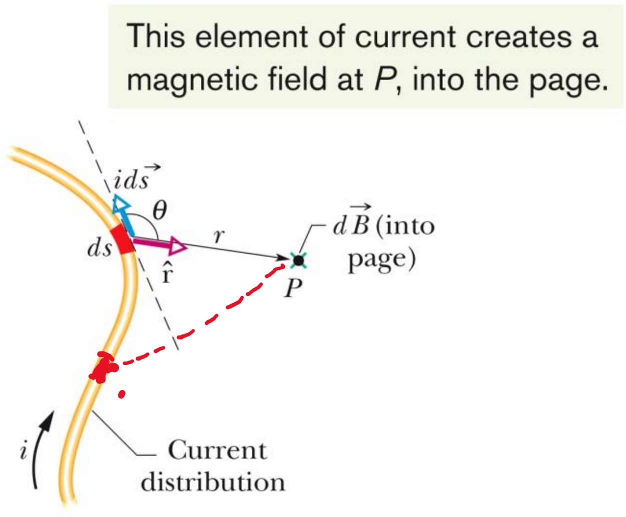
Biot Savart Law

The magnitude of the field ***dB*** produced at point P at distance *r* by a current-length element ***ds*** turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

where ϑ is the angle between the directions of ***ds*** and $\hat{\boldsymbol{r}}$, a unit vector that points from ***ds*** toward P. Symbol μ_0 is a constant, called the permeability constant, whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}.$$

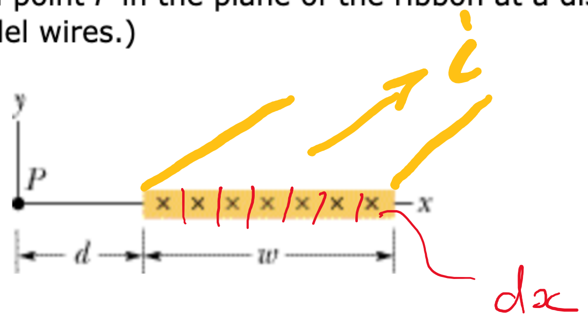


The direction of ***dB***, shown as being into the page in the figure, is that of the cross product ***ds***×$\hat{\boldsymbol{r}}$. We can therefore write the above equation containing ***dB*** in vector form as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$

This vector equation and its scalar form are known as the **law of Biot and Savart.**

The figure below shows a cross section of a long thin ribbon of width w = 4.75 cm that is carrying a uniformly distributed total current i = 6.47 μA into the page. Calculate the magnitude of the magnetic field at a point P in the plane of the ribbon at a distance d = 2.04 cm from its edge. (*Hint:* Imagine the ribbon as being constructed from many long, thin, parallel wires.)



Think.

$$B = \frac{\mu_0 i}{2\pi r}$$

Solve

$$di = i\,\frac{dx}{w}$$

$$dB = \frac{\mu_0\, di}{2\pi x} = \frac{\mu_0 i\, dx}{2\pi w\, x}$$

$$B = \int dB = \frac{\mu_0 i}{2\pi w}\int_d^{d+w} \frac{dx}{x}$$

$$B = \frac{\mu_0 i}{2\pi w}\ln\left(1 + \frac{w}{d}\right)$$

Check. $d \gg w$

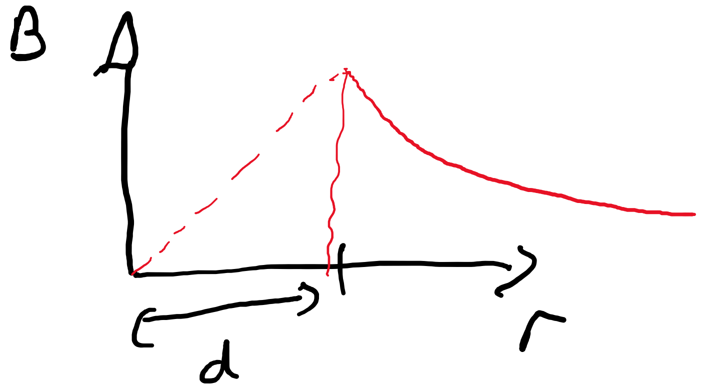
$$\ln(1+x) = x - \frac{x^2}{2} \cdots$$

$$\rightarrow B_P \rightarrow \frac{\mu_0 i}{2\pi d}.$$

Ampere's Law

- Can be expressed as: $\oint \vec{B}.d\vec{s} = \mu_o i_{enc}$
- I_{enc} is the current enclosed by the Amperian Loop

- Q: Inside a long metallic conductor, carrying a current I, where is the magnetic field B = 0
  - A. At the center
  - B. At the surface
  - C. Outside
  - D. Everywhere inside the conductor.



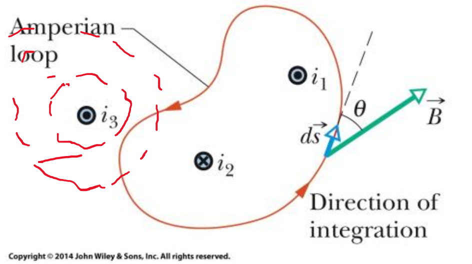
Ampere's Law

Ampere's law states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current *i* on the right side is the net current encircled by the loop.

Only the currents encircled by the loop are used in Ampere's law.



Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

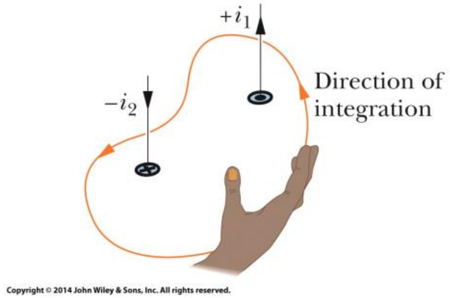
Magnetic Fields of a long straight wire with current:

$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire).}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \quad \text{(inside straight wire).}$$

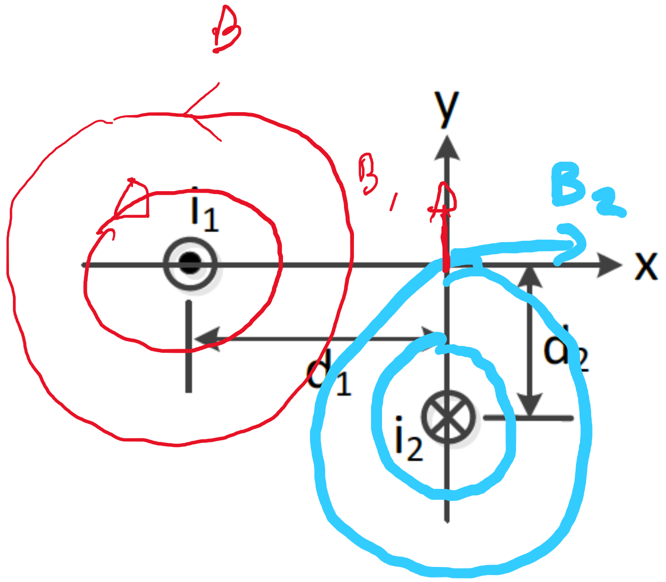
A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop.

This is how to assign a sign to a current used in Ampere's law.



Magnetic Fields due to currents

- For two long straight wires as shown, which expression describes the magnetic field at the origin?



i) $\frac{\mu_o i_2}{2\pi d_2}\,\hat{\imath}$

ii) $\frac{\mu_o i_1}{2\pi d_1}\,\hat{\jmath}$

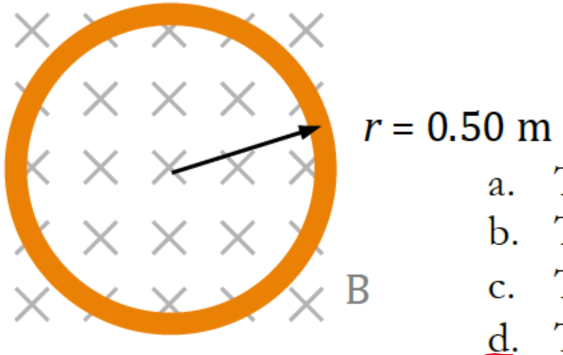
iii) $\frac{\mu_o i_2}{2\pi d_2}\,\hat{\imath} - \frac{\mu_o i_1}{2\pi d_1}\,\hat{\jmath}$

iv) $\frac{\mu_o i_1}{2\pi d_2}\,\hat{\imath} + \frac{\mu_o i_2}{2\pi d_1}\,\hat{\jmath}$

v) $\frac{\mu_o i_2}{2\pi d_2}\,\hat{\imath} + \frac{\mu_o i_1}{2\pi d_1}\,\hat{\jmath}$ ✓

Faraday's Law

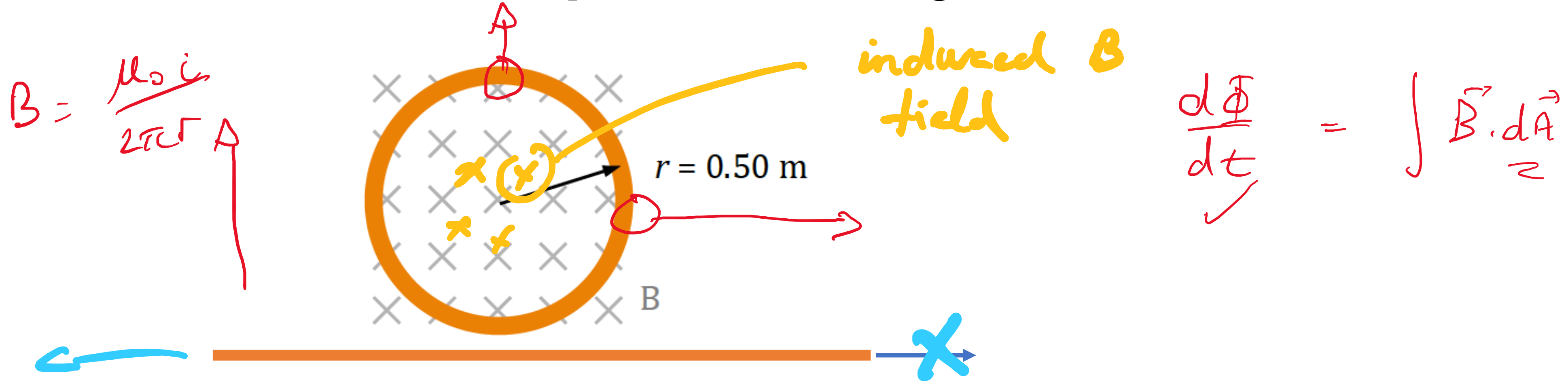
- $\mathcal{E} = -\frac{d\Phi_B}{dt}$ where the flux is given by: $\Phi_B = \int \vec{B}.d\vec{A}$
- The flux induced always opposes the change in the flux of B. However, B induced is not always opposite to B.
- Consider the loop in a constant B field with B = 4 T, which statement is true:



a. The flux through the loop is 0, and current flows in the counterclockwise direction.
b. The flux through the loop is 0, and current flows in the clockwise direction.
c. The flux through the loop is π Webers, and current flows in the clockwise direction.
d. The flux through the loop is π Webers, and current flows in the counterclockwise direction.
e. The flux through the loop is π Webers, and the current is 0.

Faraday's Law

- *Now consider the loop next to a long wire*:

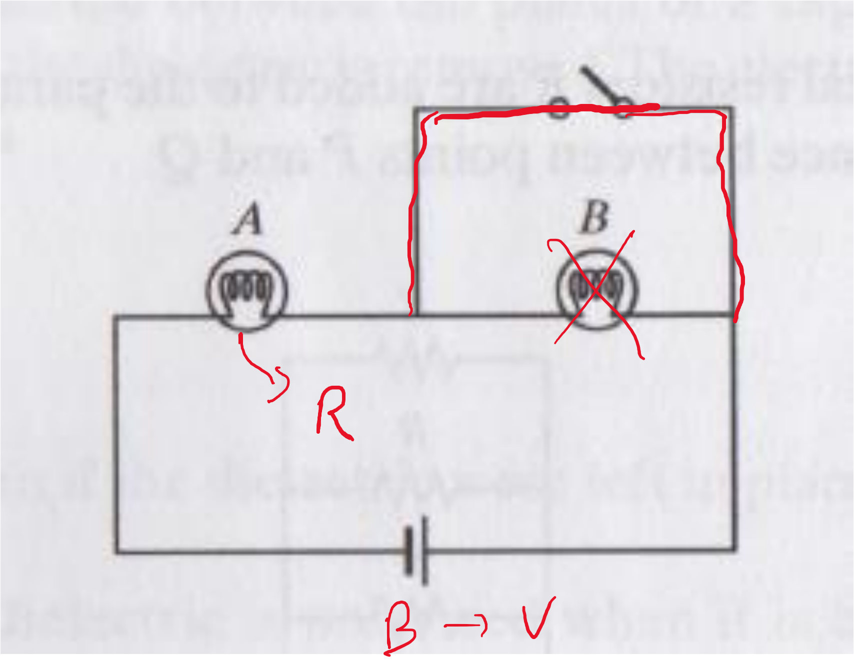


- What is the direction of the current in the loop if it is moving parallel to the wire?
- What is the direction of the current in the loop if it is moving away from the wire?

DC circuits - Concept Questions

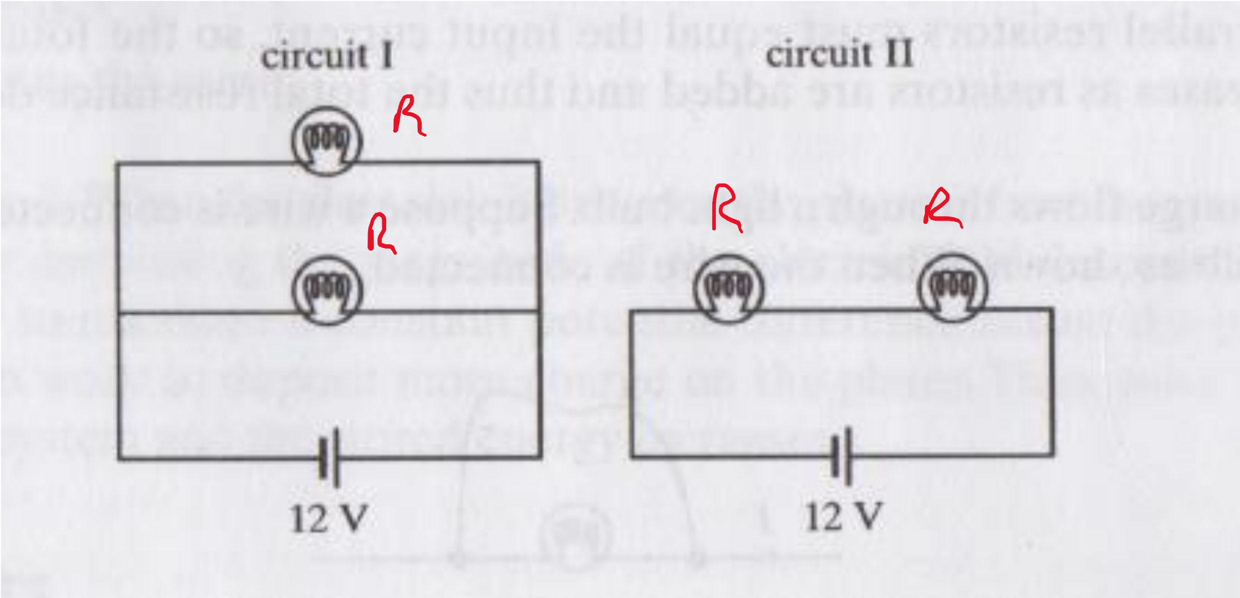
- The circuit has two identical light bulbs connected to a battery. What happens to the brightness of A and B when the switch is closed?

$P = iV$



Concept Questions

- Which circuit generates more light?

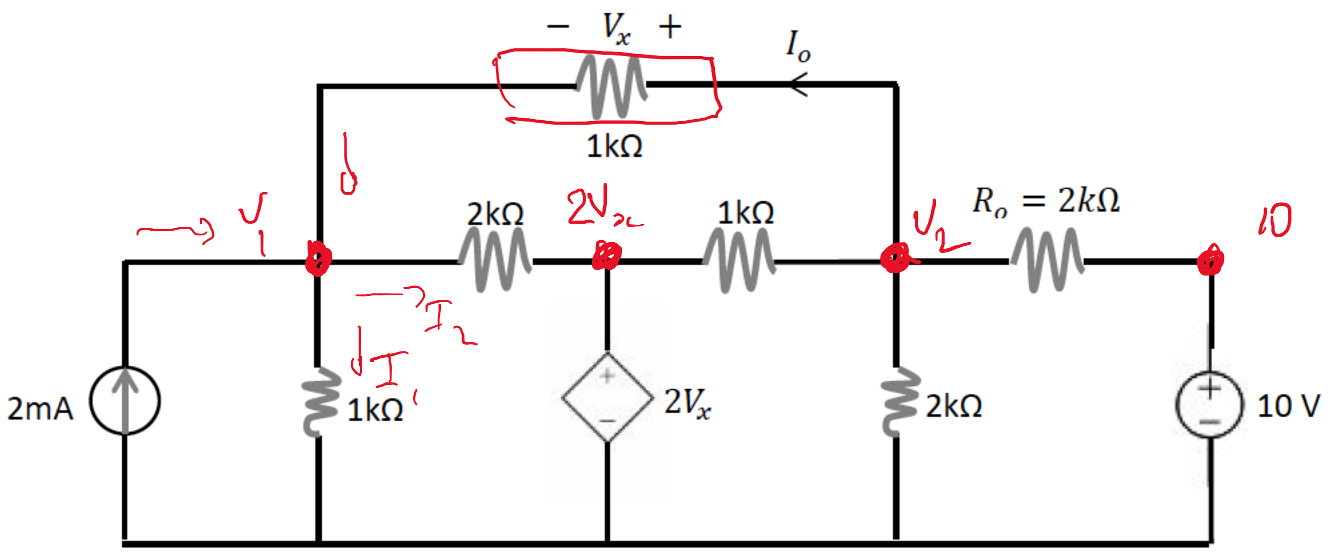


DC Circuits

1) Think
2) solve
3) check

Q3 [10 marks]

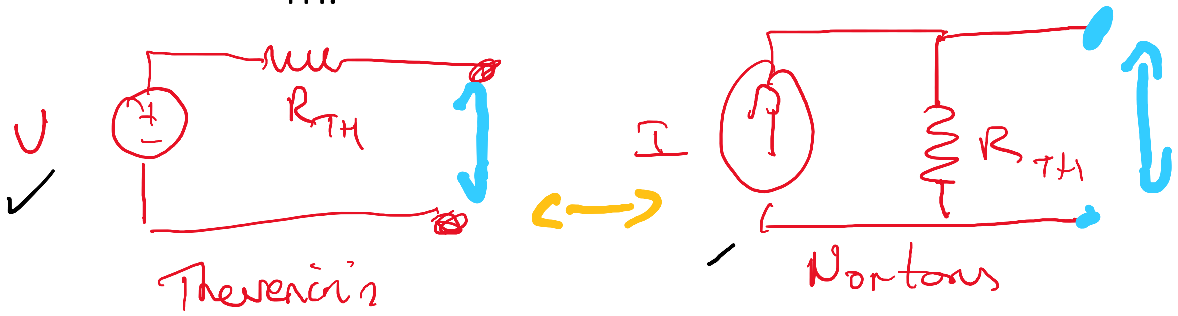
Consider the following circuit:



(a) **[7 marks]** Compute I_o using Nodal Analysis

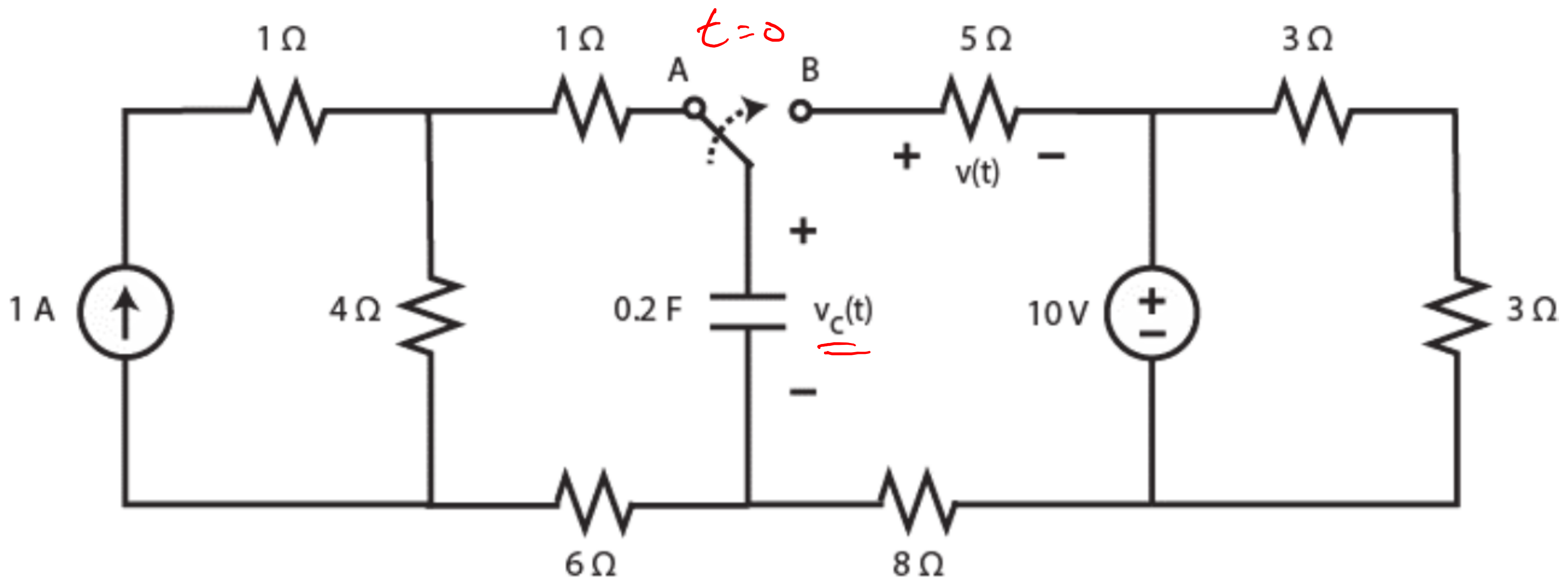
Thévenin's and Norton's

- Remove the load and find the open circuit voltage of the circuit
- To determine the R_{TH} depends on the circuit
  - Only independent sources. Replace voltage sources with short circuits and current sources with open circuits
  - Only Dependent sources. Connect an independent current or voltage source to the terminals an calculate the voltage or current. Use Ohm's Law to calculate R_{RH}
  - Both Dependent and Independent sources. Short circuit the output terminals and calculate i_{sc}. Use this with v_{oc} to calculate R_{TH}.
- For Norton's start with i_{sc}



First order circuits

- Can you find an expression for $v_c(t)$ and $v(t)$ for $t>0$



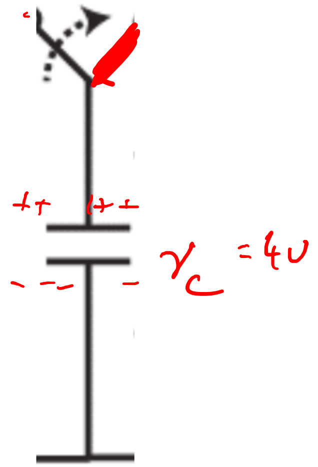
First order circuits

- Have a solution of the form: $v(t) or\ i(t) = K_1 + K_2 e^{-\frac{t}{\tau}}$
- Time constant $\tau = RC\ or\ L/R$.
- K_1 is the steady state solution. →
- K_2 is the transient solution   at $t=0$ we know $e^0 = 1$
- The current through an inductor is a continuous function
  - $i_l(t = 0^-) = i_l(t = 0^+)$ → Faraday Law
- The voltage across a capacitor is a continuous function
  - $v_c(t = 0^-) = v_c(t = 0^+)$ → charging a capacitor

First order circuits

A capacitor in a DC circuit is an open circuit

- Can you find an expression for $v_c(t)$ and $v(t)$ for t>0

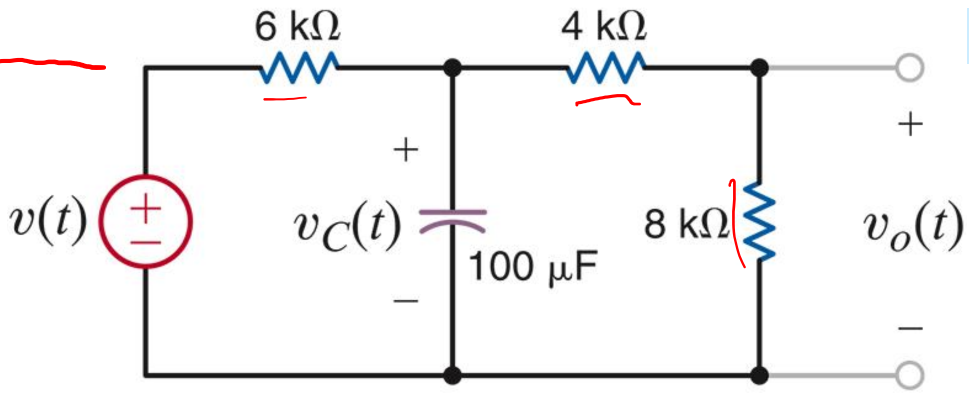


$V = iR = 5$

$V_4 = 5\left(\frac{4}{1+4}\right) = 4V = V_c$

LEARNING EXAMPLE

FIND THE OUTPUT VOLTAGE $v_O(t);\ t>0$

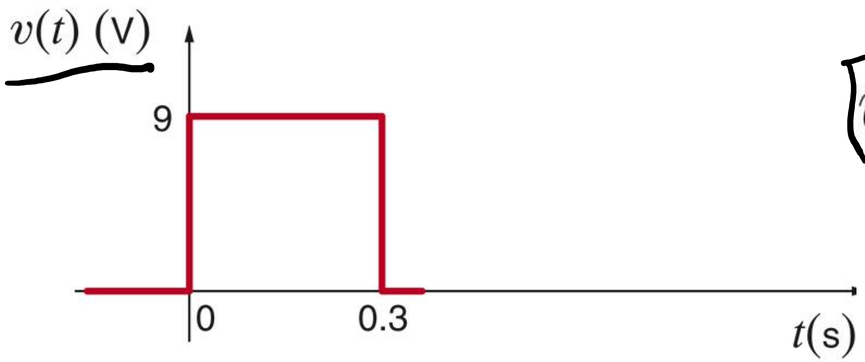


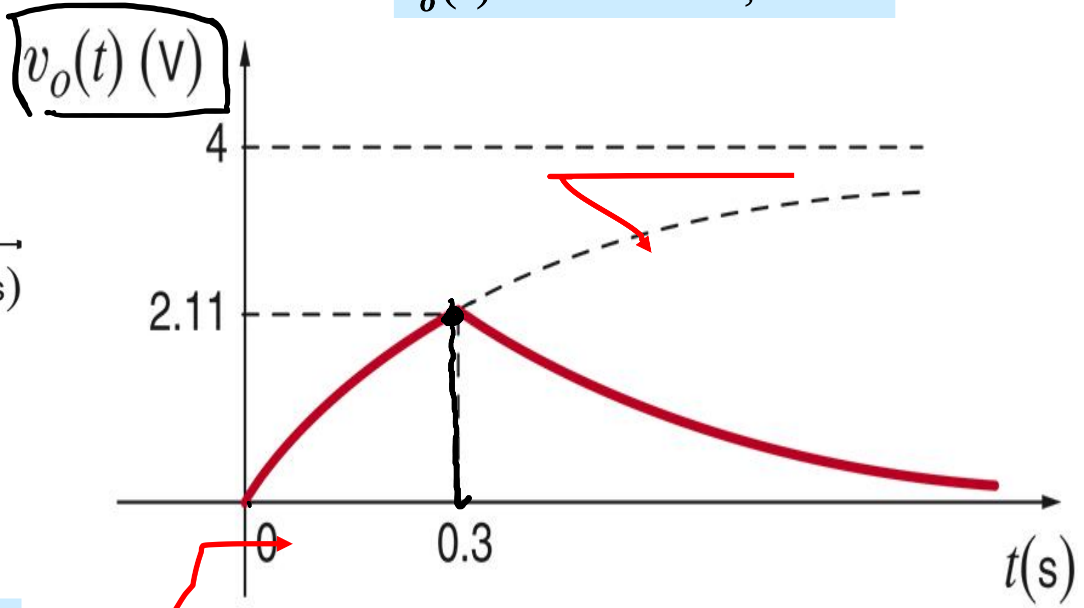
$t>0.3 \Rightarrow v(t)=0$ $t_o=0.3$ $v_o(0.3+)=4(1-e^{-\frac{0.3}{0.4}})$

$v_O(t)=K_1''+K_2''e^{-\frac{(t-0.3)}{\tau'}}$ $\tau'=0.4$

$v_o(\infty)=0 \Rightarrow K_1''=0$ $K_2''=v_o(0.3+)=2.11(V)$

$v_o(t)=2.11e^{-\frac{t-0.3}{0.4}};\ t>0.3$





$t<0 \Rightarrow v(t)=0 \Rightarrow v_O(t)=0$ $v_O(0+)=0$

$t>0 \Rightarrow v(t)=9V$

$v_o(t)=K_1'+K_2'e^{-\frac{t}{\tau}}$

$\tau=R_{TH}C=(6k\,||\,12k)\times 100\mu F=0.4s$

$v_o(\infty)=\frac{8}{10+8}(9)=K_1'$

$v_o(0+)=K_1'+K_2'=0$

$v_o(t)=4\left(1-e^{-\frac{t}{0.4}}\right)$

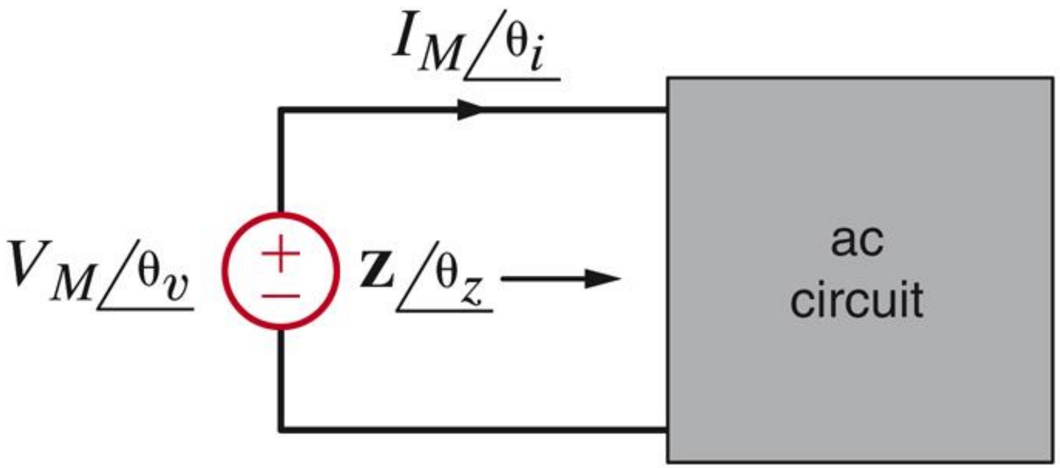
GEAUX

AC circuits

- Express voltages, and currents as Phasors
  - $I = I_m \angle \Theta_I$
  - $V = V_m \angle \Theta_v$
- Impedance is a complex number which depends on ω
  - $Z = Z_m \angle \Theta_z$ = R+jX(ω)
- For simple circuits use Ohm's law
- Apply KVL, KCL, superposition etc
- Thevenin's, Norton's etc

IMPEDANCE AND ADMITTANCE

For each of the passive components the relationship between the voltage phasor and the current phasor is algebraic. We now generalize for an arbitrary 2-terminal element.



$$Z(\omega) = R(\omega) + jX(\omega)$$
$R(\omega)$=Resistive component
$X(\omega)$=Reactive component

$$|Z| = \sqrt{R^2 + X^2}$$
$$\theta_z = \tan^{-1}\frac{X}{R}$$

(INPUT) IMPEDANCE

$$Z = \frac{V}{I} = \frac{V_M\angle\theta_v}{I_M\angle\theta_i} = \frac{V_M}{I_M}\angle(\theta_v - \theta_i) = |Z|\angle\theta_z$$

(DRIVING POINT IMPEDANCE)

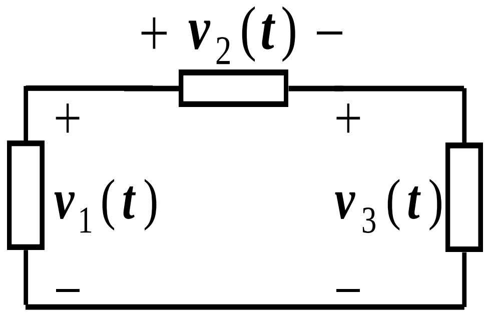
The units of impedance are OHMS.

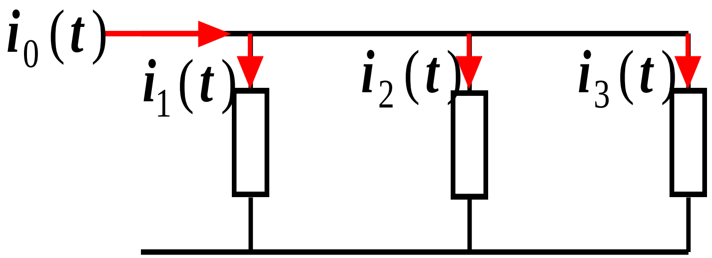
Impedance is NOT a phasor but a complex number that can be written in polar or Cartesian form. In general, its value depends on the frequency.

Element	Phasor Eq.	Impedance
R	$V = RI$	$Z = R$
L	$V = j\omega LI$	$Z = j\omega L$
C	$V = \frac{1}{j\omega C}I$	$Z = \frac{1}{j\omega C}$

GEAUX

KVL AND KCL HOLD FOR PHASOR REPRESENTATIONS





KVL : $v_1(t) + v_2(t) + v_3(t) = 0$

KCL : $-i_0(t) + i_1(t) + i_2(t) + i_3(t) = 0$

$$v_i(t) = V_{Mi} e^{j(\omega t + \theta_i)},\ i = 1,2,3$$

$$i_k(t) = I_{Mk} e^{j(\omega t + \phi_k)},\ k = 0,1,2,3$$

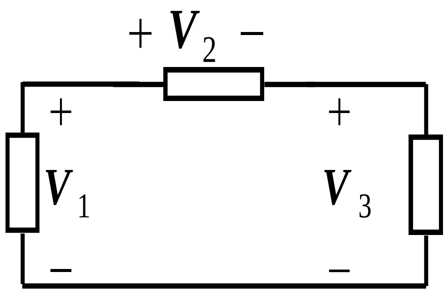
KVL : $(V_{M1}e^{j\theta_1} + V_{M2}e^{j\theta_2} + V_{M3}e^{j\theta_3})e^{j\omega t} = 0$

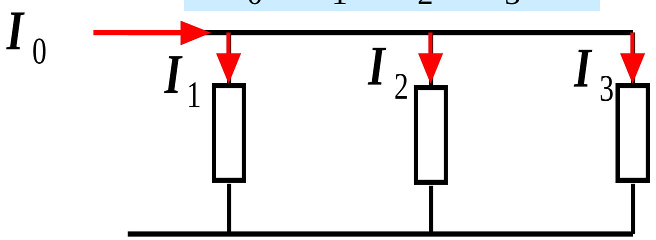
In a similar way, one shows ...

$$V_{M1}\angle\theta_1 + V_{M2}\angle\theta_2 + V_{M3}\angle\theta_3 = 0$$

$V_1 + V_2 + V_3 = 0$ Phasors!

$-I_0 + I_1 + I_2 + I_3 = 0$

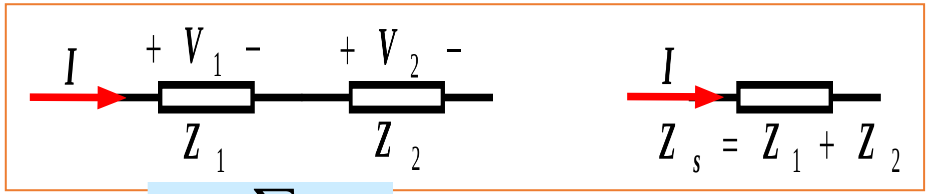




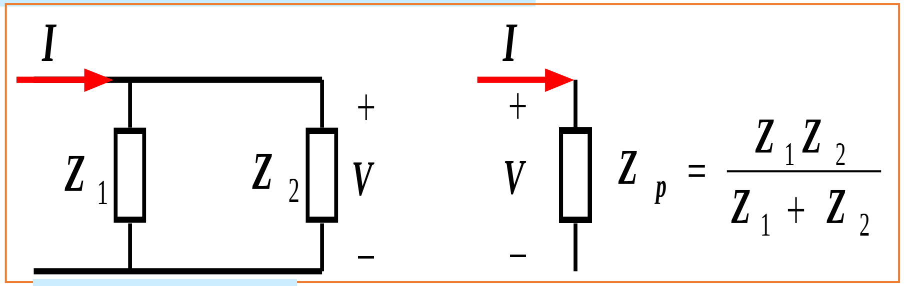
The components will be represented by their impedances and the relationships will be entirely algebraic!!

GEAUX

SPECIAL APPLICATION:
IMPEDANCES CAN BE COMBINED USING THE SAME RULES DEVELOPED FOR RESISTORS

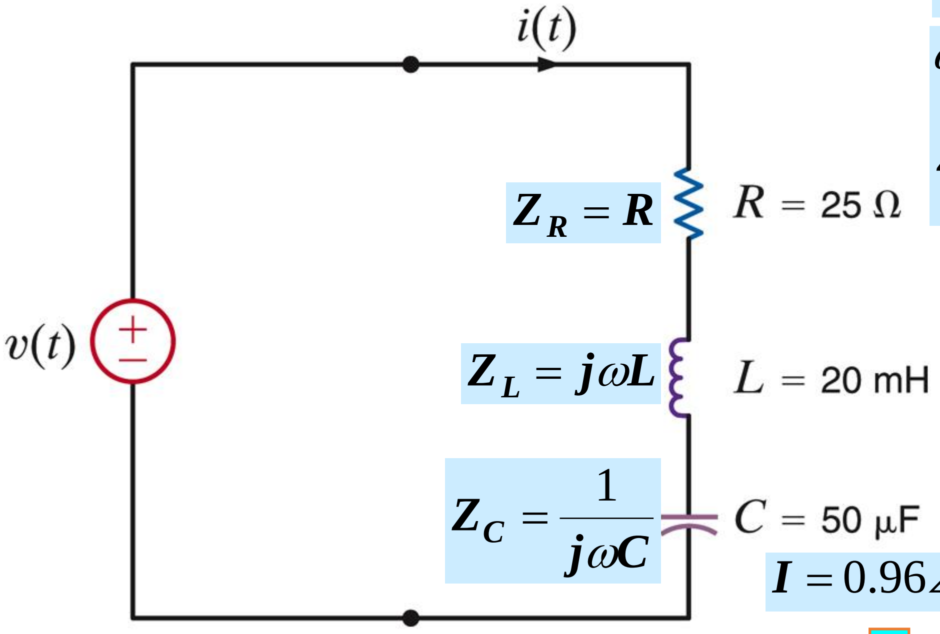


$$Z_s = \sum_k Z_k$$



$$\frac{1}{Z_p} = \sum_k \frac{1}{Z_k}$$

LEARNING EXAMPLE



$f = 60 Hz, v(t) = 50\cos(\omega t + 30°)$

Compute equivalent impedance and current

$\omega = 120\pi, V = 50\angle 30°, Z_R = 25\Omega$

$$Z_L = j120\pi \times 20 \times 10^{-3}\Omega,\ Z_C = \frac{1}{j120\pi \times 50 \times 10^{-6}}$$

$$Z_L = j7.54\Omega,\ \ Z_C = -j53.05\Omega$$

$$Z_s = Z_R + Z_L + Z_C = 25 - j45.51\Omega$$

$$I = \frac{V}{Z_s} = \frac{50\angle 30°}{25 - j45.51}(A) = \frac{50\angle 30°}{51.93\angle -61.22°}(A)$$

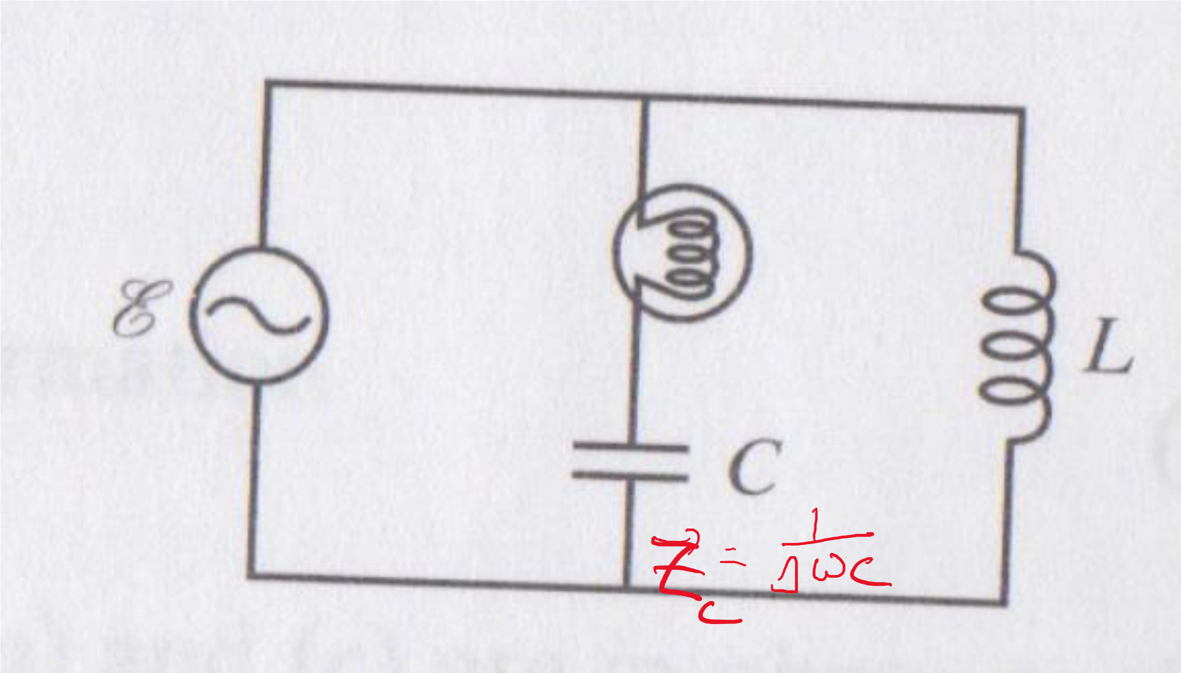
$$I = 0.96\angle 91.22°(A) \Rightarrow i(t) = 0.96\cos(120\pi t + 91.22°)(A)$$

GEAUX

An AC circuit to consider

- At which frequencies does the light bulb glow brightest?

- Low Frequencies
- High Frequencies ✓
- At $\omega = \frac{1}{\sqrt{LC}}$

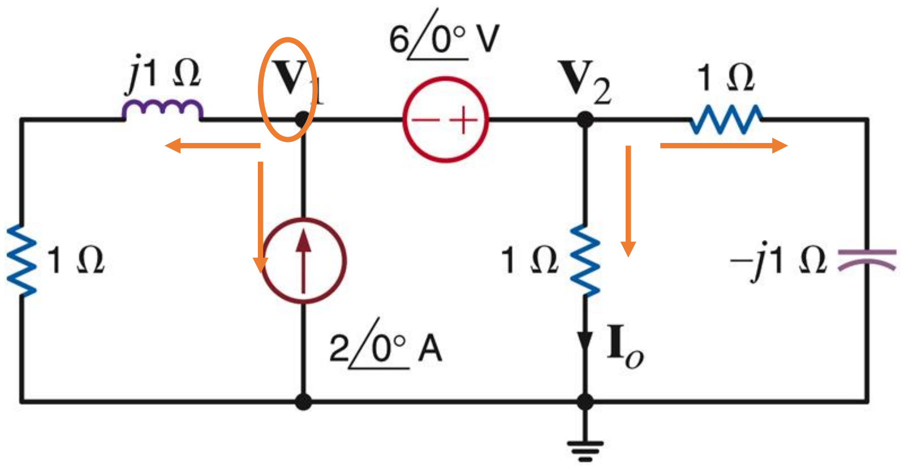


$Z_L = j\omega L$

ANALYSIS TECHNIQUES

PURPOSE: TO REVIEW ALL CIRCUIT ANALYSIS TOOLS DEVELOPED FOR RESISTIVE CIRCUITS; I.E., NODE AND LOOP ANALYSIS, SOURCE SUPERPOSITION, SOURCE TRANSFORMATION, THEVENIN'S AND NORTON'S THEOREMS.

COMPUTE I_0



1. NODE ANALYSIS

$$\frac{V_1}{1+j1} - 2\angle 0^\circ + \frac{V_2}{1} + \frac{V_2}{1-j1} = 0$$

$$V_1 - V_2 = -6\angle 0^\circ$$

$$I_0 = \frac{V_2}{1}(A)$$

$$\frac{V_2 - 6\angle 0^\circ}{1+j1} - 2\angle 0^\circ + V_2 + \frac{V_2}{1-j1} = 0$$

$$V_2\left[\frac{1}{1+j1} + 1 + \frac{1}{1-j1}\right] = 2 + \frac{6}{1+j1}$$

$$V_2 \frac{(1-j1)+(1+j1)(1-j1)+(1+j1)}{(1+j1)(1-j1)} = \frac{2(1+j1)+6}{1+j1}$$

$$V_2 \frac{4}{1-j} = 8 + j2$$

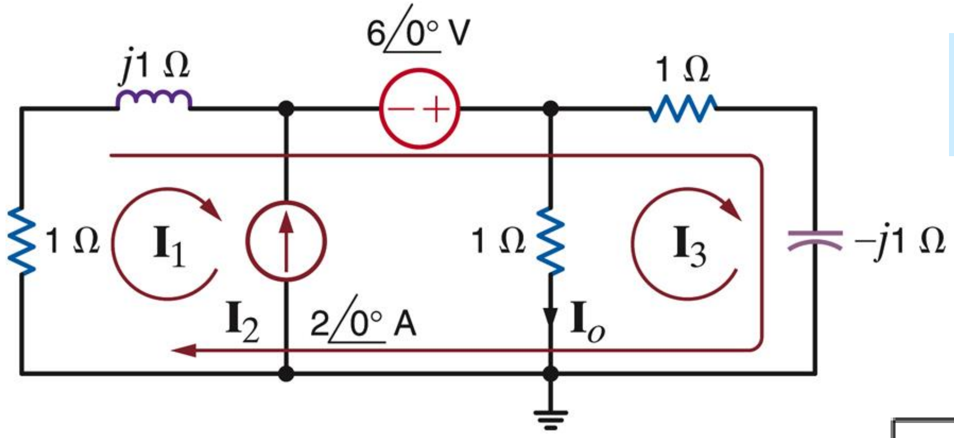
$$V_2 = \frac{(4+j)(1-j)}{2}$$

$$I_0 = \left(\frac{5}{2} - j\frac{3}{2}\right)(A)$$ $$I_0 = 2.92\angle -30.96^\circ$$

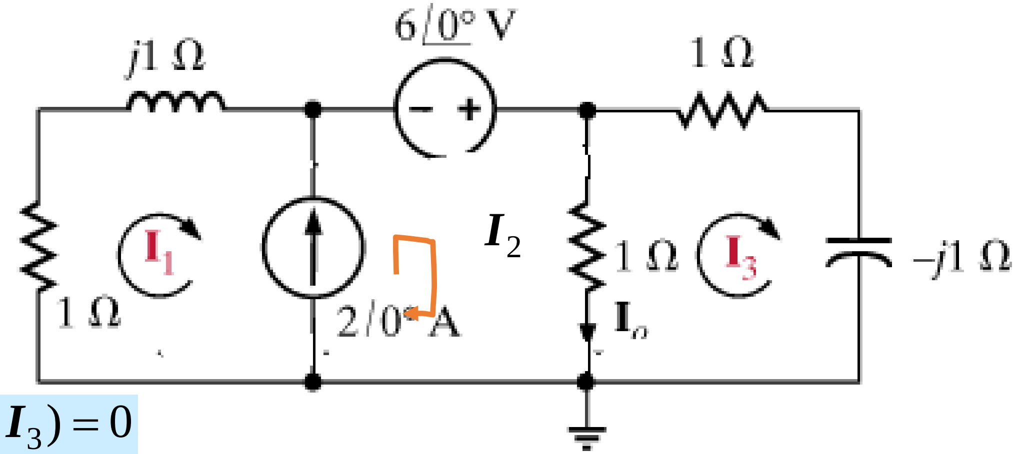
NEXT: LOOP ANALYSIS

GEAUX

2. LOOP ANALYSIS



ONE COULD ALSO USE THE SUPERMESH TECHNIQUE



SOURCE IS NOT SHARED AND Io IS DEFINED BY ONE LOOP CURRENT

$I_0 = -I_3$

LOOP 1: $I_1 = -2\angle 0°$

LOOP 2: $(1+j)(I_1+I_2) - 6\angle 0° + (1-j)(I_2+I_3) = 0$

LOOP 3: $(1-j)(I_2+I_3) + I_3 = 0$

MUST FIND I_3

$2I_2 + (1-j)I_3 = 6 - (1+j)(-2)$ /* $(1-j)$

$(1-j)I_2 + (2-j)I_3 = 0$ /* (-2)

$\left((1-j)^2 - 2(2-j)\right)I_3 = (1-j)(8+2j)$

$I_3 = \frac{10-6j}{-4}$ $I_0 = \frac{5}{2} - \frac{3}{2}j(A)$

CONSTRAINT: $I_1 - I_2 = -2\angle 0°$

SUPERMESH: $(1+j)I_1 + 6\angle 0° + (I_2 - I_3) = 0$

MESH 3: $(I_3 - I_2) + (1-j)I_3 = 0$

$I_0 = I_2 - I_3$