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Surname ______________________________

First Name ______________________________

Student Number ______________________________

University of Toronto
Faculty of Applied Science and Engineering

FINAL EXAMINATION, APRIL 2018
SECOND YEAR - PROGRAM 6
ChE210S HEAT AND MASS TRANSFER

April 26, 2018; 9:30 am to 12:00 noon
EXAMINER: Y.-L. CHENG

Instructions:

1. Answer all questions.
2. Do all work on these sheets.
3. Exam Type A: closed book examination, no aids are permitted other than the information printed on the examination paper. An aid sheet is provided as part of the examination paper.
4. Calculate Type 2: non-programmable calculators are allowed.
5. Marks for each problem and parts of problems are indicated in brackets.

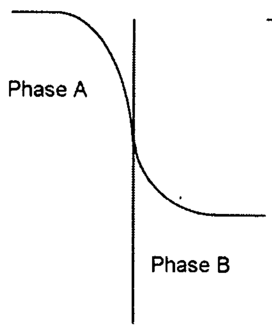
Problem	Marks Earned
1	/15
2	/25
3	/20
4	/20
5	/20
Total	/100

Problem 1 [15 Marks Total]

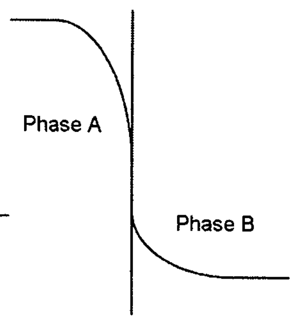
Provide brief answers to each of the questions below.

1(a) [5 Marks]

The temperature and concentration profiles in two contacting phases in which heat and mass transfer are occurring are shown. Explain why the temperature profile is continuous across the interface while the concentration profile is discontinuous.



Temperature Profile



Concentration Profile

1(b) [5 Marks]

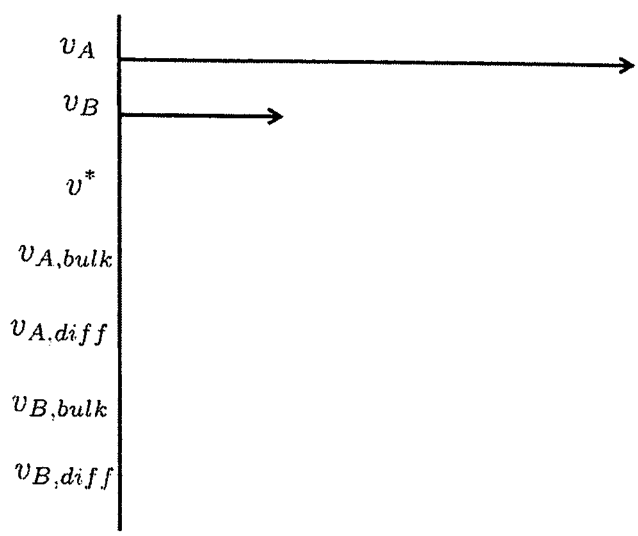
Consider three rectangular slabs of thickness L made up of materials A, B and C with properties shown. Each slab is initially at a uniform temperature T_L. At time $t = 0$, the surface at $x = 0$ is subjected to a step temperature change such that for all $t \geq 0$, $T = T_o$ at $x = 0$; T remains T_L at $x = L$. We expect the $T(x, t)$ semi-infinite medium solution to be applicable to the finite slab at short times. Provide an inequality that gives the rank order of the time duration for which you expect the semi-infinite medium solution to be valid. E.g. $A > B > C$ means the solution will be valid for medium A for the longest amount of time.

	$k\ (W \cdot m^{-1} \cdot K^{-1})$	$\rho C_p\ (J \cdot m^{-3} K^{-1})$	$\alpha\ (m^2/s)$
A	0.3	3.3×10^4	9×10^{-6}
B	0.6	10×10^4	6×10^{-6}
C	0.9	30×10^4	3×10^{-6}

1(c) [5 Marks]

Ficks Law of Diffusion $N''_{Ax} = cDy_A + y_A \sum_{i=1}^{n} N''_{ix}$ says the absolute flux of a component "A" consists of a bulk component and a diffusive component. The absolute velocities of "A" and "B" in an equimolr mixture of the two are as shown. Draw arrows of the appropriate **magnitude** and **direction** to indicate the following quantities:

1. $v*$ - the molar average velocity of the mixture
2. $v_{A,bulk}$ - velocity of "A" due to bulk mixture motion
3. $v_{A,diff}$ - velocity of A due to diffusive motion
4. $v_{B,bulk}$ - velocity of B due to bulk mixture motion
5. $v_{B,diff}$ - velocity of "B" due to diffusive motion



Problem 2 [25 Marks Total]

We have discussed convective mass transfer coefficients k_c in great detail in this course. A simplistic alternative approach of modelling convective mass transfer is the "film theory" in which it is imagined that a hypothetical stagnant (no bulk flow) fluid film of thickness L exists next to a solid surface. In this scenario, the concentration at the solid/fluid surface ($x = 0$) is $C_A = C_{AS}$, and the concentration at the far edge of hypothetical film $x = L$ is the same as the concentration of "A" in the bulk fluid, or $C_A = C_{A\infty}$. The stagnant film thickness L is a model parameter fitted to experimental data such that diffusion through a stagnant film of thickness "L" gives the same surface molar flux of "A" as experimentally measured.

2(a) [10 Marks]

Obtain an expression for the steady state diffusional flux in the hypothetical stagnant film of thickness L. The diffusion coefficient of "A" in the fluid is D, and the surface and bulk concentrations are C_{AS} and $C_{A\infty}$, respectively. "A" can be considered to be dilute in the mixture.

[Problem 2, continued]

2(b) [5 Marks]

If the convective mass transfer coefficient under some specified flow condition is k_c, what is L?

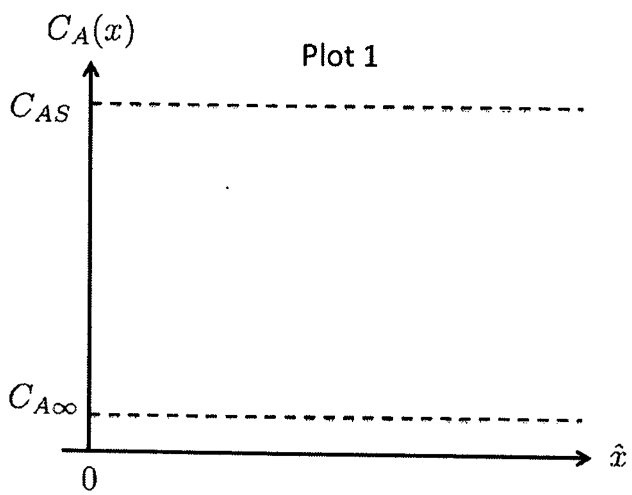
2(c) [10 Marks]

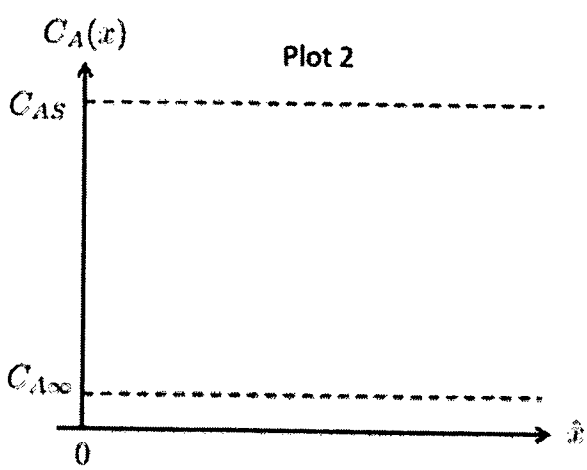
In the graphs shown, plot the concentration profiles specified. Label stagnant film thickness(es) where appropriate. Pay particular attention to the relative slopes of your graphs at the $x = 0$ surface.

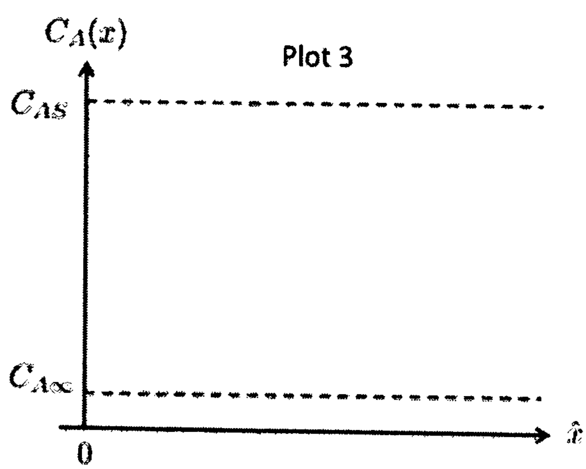
Plot 1: (a) actual concentration profile (with flow), and (b) the hypothetical concentration profile in an equivalent stagnant film.

Plot 2: actual concentration profiles for two flow situations: (a) high flow, and (b) low flow

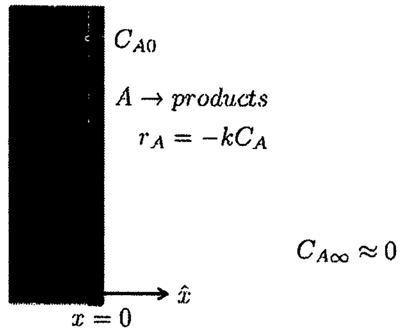
Plot 3: hypothetical film theory concentration profiles that model two flow situations: (a) high flow, and (b) low flow







[Problem 3, 20 marks]



We wish to extract A from a solid using a liquid solvent in which A dissolves. It is known that the overall dissolution/extraction is mass transfer limited. It has been suggested that by making "A" chemically react in the liquid (eg by changing pH) will enhance mass transfer. As shown in the figure: (a) At the $x = 0$ solid/liquid interface, the concentration of A in the liquid is C_{A0}, (b) in the bulk liquid, $C_A = C_{A\infty} \approx 0$, and (c) A reacts in the liquid at a rate per unit volume of $r_A = -kC_A$ $[moles\ A\,/(m^3 \cdot s^1)]$ where k is a reaction rate constant.

[3(a) 12 Marks]

Derive an expression for the steady state concentration profile of A in the solvent $C_A(x)$. Model this problem as a stagnant problem, i.e. you may assume there is no flow, and that "A" is dilute in the liquid solution. [Note: there is a clear boundary condition at $x = 0$, think carefully about what the second boundary condition should be.]

[Problem 3, continued]

[3(b) 4 Marks]

Obtain an expression for the convective mass transfer coefficient k_c.

[3(c) 4 Marks]

Provide a physical explanation for why the chemical reaction would enhance mass transfer and the overall extraction rate.

Problem 4 [20 Marks]

4(a) [10 Marks]

A thin flat plate that has dimensions of $0.2\ m \times 0.2\ m$ and is oriented parallel to an atmospheric airstream having a velocity of $40\ m/s$. The air temperature T_∞ is 20^oC, while the plate is maintained at $T_s = 120^oC$. The air flows over both the top and bottom surfaces of the plate, and measurement of the drag force on the plate gives a value of $0.075\ N$. What is the total rate of heat transfer from the plate to the air?

Properties of air under these conditions (air at 1 atm and 70^oC film temperature):
$\rho = 1.018\ kg/m^3, \quad C_p = 1009J/(kg \cdot K), \quad Pr = 0.70, \quad \nu = 2.02.2 \times 10^{-5} m^2/s.$

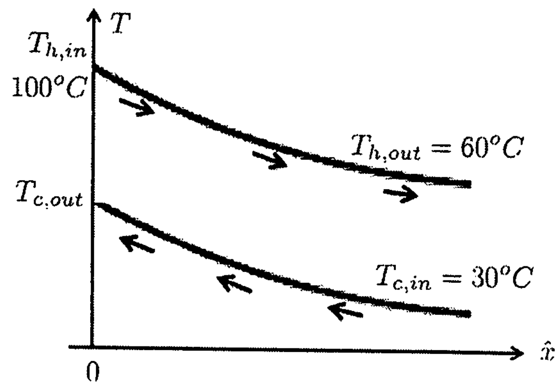
[Problem 4, continued]

4(b) [10 Marks]

A gas X flows over a flat plate of length $0.1\ m$ along the direction of flow. The boundary layer remains laminar along the entire plate, and the average heat transfer coefficient is measured to be $h_L = 25\ W/(m^2 \cdot K)$. The plate is then impregnated with a liquid Y and subjected to the same flow conditions. Given the following thermophysical properties, what is the average convective mass transfer coefficient k_{cL} along the plate?

	$\nu\ [m^2/s]$	$k\ [W/(m \cdot K)]$	$\alpha\ [m^2/s]$	$D\ [m^2/s]$
Gas X	21×10^{-6}	0.030	29×10^{-6}	
Liquid Y	3.75×10^{-7}	0.665	1.65×10^{-7}	
Vapor Y	4.25×10^{-5}	0.023	4.55×10^{-5}	
Binary mixture of gas X - vapor Y				2.92×10^{-5}

Problem 5 [20 marks]



Consider a concentric tube heat exchanger for cooling lubricating oil that consists of a thin-walled inner tube in which cooling water flows, and an outer tube- with oil flowing in the annular region between the two tubes. The overall heat transfer coefficient is U, and the total contact area is A for heat exchanger length of L.

[5(a) 10 Marks]

Show that the total rate of heat transfer from the hot stream to the cold streams in the heat exchanger is given by:

$$q = UA\Delta T_{lm} = UA\left[\frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}\right] \quad \text{where} \quad \Delta T_{lm} = \left[\frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}\right]$$ is the log mean temperature difference.

The driving force for heat transfer for hot to cold streams is $\Delta T = T_h - T_c$, and ΔT_1 and ΔT_2 are the the values of the driving force at the two ends of the heat exchanger.

[Problem 5, continued]

[5(b) 10 Marks]

The exchanger operates in counterflow with an overall heat transfer coefficient U of 60 $W/(m^2 \cdot K)$. The mass flow rates of the hot and cold water streams are the same at $\dot{m}_h = \dot{m}_c = 0.1kg/s$. The inlet and outlet temperatures of the oil stream are $100°C$ and $60°C$, respectively. The inlet temperature of the cooling water is $30°C$. The inner column diameter is 25 mm, and the outer column diameter is 45 mm.

Neglect heat losses to the surrounding, and determine (i) the total rate of heat transfer between the two streams, (ii) $T_{c,out}$ the outlet temperature of the cold water stream, and (iii) the length of the heat exchanger.

Data: Water: $C_p = 4200\ J/(kg \cdot K)$, Oil: , $C_p = 1900\ J/(kg \cdot K)$

Convective Transport: Definitions and Analogy

$$\text{Coefficient of Friction: } C_f \equiv \frac{\tau_o}{\frac{1}{2}\rho v_\infty^2}$$

$$\text{Nusselt Number: } Nu = \frac{hL}{k}$$

$$\text{Sherwood Number: } Sh = \frac{k_c L}{D}$$

$$\text{Chilton-Colburn Analogy: } \frac{C_f}{2} = \frac{h}{\rho C_p v_\infty} Pr^{2/3} = \frac{k_c}{v_\infty} Sc^{2/3}$$

Heat Diffusion Equation (constant k) and Fourier's Law of Conduction

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho C_p} \qquad \text{and} \qquad \vec{q''} = -k\vec{\nabla} T$$

Cartesian Coordinates

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{\dot{q}}{\rho C_p}$$

$$q''_x = -k\frac{\partial T}{\partial x} \qquad q''_y = -k\frac{\partial T}{\partial y} \qquad q''_z = -k\frac{\partial T}{\partial z}$$

Cylindrical Coordinates

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{\dot{q}}{\rho C_p}$$

$$q''_r = -k\frac{\partial T}{\partial r} \qquad q''_\phi = -\frac{k}{r}\frac{\partial T}{\partial \phi} \qquad q''_z = -k\frac{\partial T}{\partial z}$$

Diffusion Equation (constant D and c) and Fick's Law of Diffusion

$$\vec{N''_A} = -cD\vec{\nabla} x_A + x_A \sum \vec{N''_i} \qquad \text{(molar flux of A)}$$

In stationary medium or if "A" is dilute, and c = constant: Diffusion equation and Fick's Law:

$$\frac{\partial C_A}{\partial t} = D\nabla^2 C_A + \dot{N}_A \qquad \text{and} \qquad \vec{N''_A} = -D\vec{\nabla} C_A$$

$$\frac{\partial C_A}{\partial t} = D\nabla^2 C_A + \dot{N}_A$$

Cartesian Coordinates

$$\frac{\partial C_A}{\partial t} = D \left[\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right] + \dot{N}_A$$

$$N''_{Ax} = -D\frac{\partial C_A}{\partial x} \qquad N''_{Ay} = -D\frac{\partial C_A}{\partial y} \qquad N''_{Az} = -D\frac{\partial C_A}{\partial z}$$

Cylindrical Coordinates

$$\frac{\partial C_A}{\partial t} = D \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 C_A}{\partial \phi^2} + \frac{\partial^2 C_A}{\partial z^2} \right] + \dot{N}_A$$

$$N''_{Ar} = -D\frac{\partial C_A}{\partial r} \qquad N''_{A\phi} = -\frac{D}{r}\frac{\partial C_A}{\partial \phi} \qquad N''_{Az} = -D\frac{\partial C_A}{\partial z}$$